NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

EVALUATION OF THE EFFECTS OF ROLL ANGLE ON THE NATURAL CONVECTION HEAT TRANSFER FROM AN ARRAY OF CYLINDRICAL SPINES

by

Joseph E. McClanahan

December 1984

Thesis Advisor: A. D. Kraus

Approved for public release; distribution unlimited

SECURITY CLASSIFICATION OF THIS PAGE (*When Data Entered*)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (*and Subtitle*) Evaluation of the Effects of Roll Angle on the Natural Convection Heat Transfer from an Array of Cylindrical Spines		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis December 1984
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Joseph E. McClanahan		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93943		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93943		12. REPORT DATE December 1984
		13. NUMBER OF PAGES 50
14. MONITORING AGENCY NAME & ADDRESS(*if different from Controlling Office*)		15. SECURITY CLASS. (*of this report*) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (*of this Report*)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (*of the abstract entered in Block 20, if different from Report*)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (*Continue on reverse side if necessary and identify by block number*)

natural convection, finned array, cylindrical spines, heat transfer, roll angle enhancement, Nusselt number, Rayleigh number

20. ABSTRACT (*Continue on reverse side if necessary and identify by block number*)

Experimental data has been obtained that predicts the effect of roll angle upon the natural convection heat transfer from an array of cylindrical spines or pin fins. A correlation is presented that permits evaluation of the Nusselt Modulus as a function of the Rayleigh Modulus and the roll angle and it shows that maximum heat transfer is obtained at roll angles between 20 and 30 degrees.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

S/N 0102-LF-014-6601

SECURITY CLASSIFICATION OF THIS PAGE (*When Data Entered*)

Evaluation of the Effects of Roll Angle on the Natural Convection Heat Transfer from an Array of Cylindrical Spines

by

Joseph E. McClanahan
Lieutenant Commander, United States Navy
B.S., Auburn University, 1970

Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
December 1984

ABSTRACT

Experimental data has been obtained that predicts the effect of roll angle upon the natural convection heat transfer from an array of cylindrical spines or pin fins. A correlation is presented that permits evaluation of the Nusselt Modulus as a function of the Rayleigh Modulus and the roll angle and it shows that maximum heat transfer is obtained at roll angles between 20 and 30 degrees.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE

b	height of spine (m)
c_p	specific heat of surrounding fluid (J/kg·K)
d	diameter of spine (m)
dq	differential amount of heat (w)
$f_1(x)$	cross sectional area function for spine (m)
$f_2(x)$	profile function for spine (m)
$f_3(x)$	perimeter function for spine (m)
g	gravitational acceleration (m/s^2)
Gr	Grashof number (dimensionless)
h	heat transfer coefficient ($w/m^2 \cdot K$)
k	thermal conductivity ($w/m \cdot K$)
k_f	thermal cond. of surrounding fluid ($w/m \cdot K$)
k_I	thermal cond. of insulation ($w/m \cdot K$)
k_S	thermal cond. of spine material ($w/m \cdot K$)
m	fin heat transfer parameter (1/m)
Nu	Nusselt number (dimensionless)
Pr	Prandtl number (dimensionless)
q	heat flow into spine at base (w)
Ra	Rayleigh number (dimensionless)
Re	Reynolds number (dimensionless)
t, T_S	temperature of spine (fin) surface (K)

t_s, T_∞	temperature of surrounding fluid (K)
$Z(\phi)$	enhancement factor a function of the roll angle

GREEK SYMBOLS

α	a combination of variables defined by eqn (A.9)
β	volumetric coefficient of thermal expansion (1/K)
δ	diameter of spine (m)
ϕ	roll angle (degrees)
θ	temperature excess (K)
θ_b	temperature excess at spine base (K)
θ_a	temperature excess at spine tip (K)
μ	dynamic viscosity of surrounding fluid (kg/m.s)
ν	kinematic viscosity of surrounding fluid (m^2/s)

ACKNOWLEDGEMENTS

The author wishes to express his sincere appreciation to his wife Debra whose understanding and willingness to accomplish "everything else" made the difficult times endurable. He also wishes to thank Professor Allan D. Kraus for his patience and guidance in conducting the experimental portion of the work and in analyzing the results. An additional thank you is in order to Mr. Tom McCord and the gentlemen of the machine shop whose willingness and ability in fabricating the test rig is sincerely appreciated.

I. INTRODUCTION

Electronically steered phased array antennae frequently exhibit degraded performance due to the lack of proper heat transfer between the interior crystalline materials and the environment. This causes higher than optimum temperature environments for the crystalline materials and a subsequent loss of reliability[1].

One possible solution to such a problem considers a surface augmentation through the use of pin fins (or spines) to assist in the dissipation of the heat.

The problem is more severe when the installation is stationary so that the dissipation from the spines is in the natural convection mode and, because of this, the focus of this investigation is in the area of natural convection heat transfer and methods of enhancement in this domain.

Convective heat transfer from horizontal cylinders has been the subject of extensive studies which were summarized by Morgan[2]. In addition, studies of heat exchanger tube bundles are summarized by Incropera and and DeWitt[3]. However, after a considerable search of the literature, only a small amount of experimental data and analytical correlations dealing with natural convection in an array of horizontal cylinders, has been found.

Intuitively, one feels that the orientation of the array must have an effect on the heat transfer but apparently no known experimental correlations have been formulated. Thus, the focus of this thesis is in the effect of roll angle on the heat transfer rates from an array of cylindrical spines.

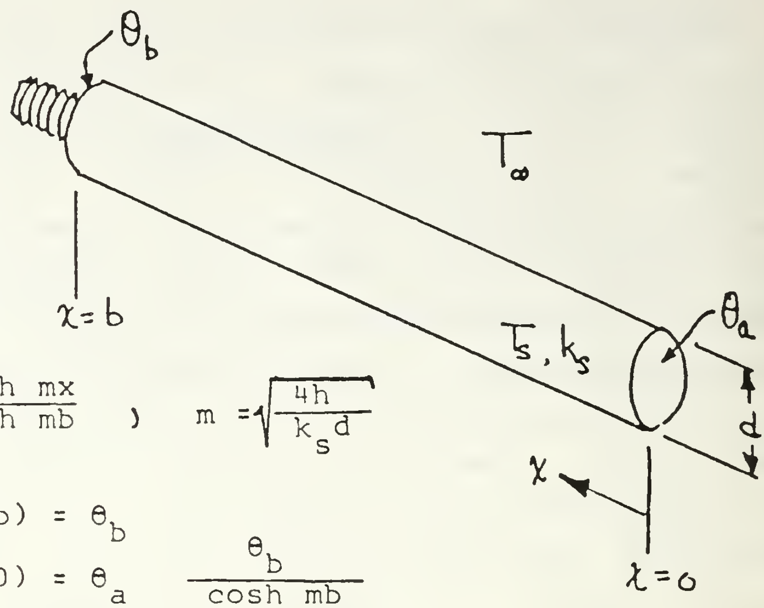
Additional impetus for investigations in similar areas with cylindrical spine arrays derives from the electronics industry where such cooling arrangements are used in the external cooling of dense integrated circuit arrangements.

Dimensionless correlations using dimensionless parameters are usually developed for presentation of experimental results and this allows for possible application to other systems. In developing the solution for the heat transfer across a boundary layer, it has been shown by Schlichting[4], and others [3,5], that the Nusselt number (Nu) is a proper dimensionless parameter for natural or forced convection. In forced convection, the Nusselt number is a function of the Reynolds number (Re) and the Prandtl number (Pr). However, in pure natural convection where the buoyancy is the only driving force, the fluid velocity is determined entirely by the quantities contained in the Grashof number (Gr) which is used instead of the Reynolds number to establish dynamic similarity. The Grashof number is developed quite thoroughly by Kreith and others [3,5] and quickly appears in a dimensional analysis.

In addition, for gases the Prandtl number (Pr) is nearly constant so that the Nusselt Modulus (Nu_d) can be considered to be a function of the product of the Grashof number and the Prandtl number. This product is defined as the Rayleigh or number (Ra) and is discussed to further detail in the following paragraphs.

In order to establish the heat transfer coefficient and to develop a correlation of the data with dimensionless parameters, relationships for the heat transfer coefficient and the Nusselt number were established and measurements were taken which permitted its calculation. A summary of the derivation is provided in Figure 1.1 and a detailed derivation is provided in Appendix A. The relation holds for the condition that no heat transfer occurs at the fin tip. Such a condition can be effectively achieved by insulating the tip.

After calculating the heat transfer coefficient the Nusselt number was then determined from the relation:



$$\theta(x) = \theta_b \frac{\cosh mx}{\cosh mb} \quad , \quad m = \sqrt{\frac{4h}{k_s d}}$$

$$\text{at } x = b, \ \theta(b) = \theta_b$$

$$\text{at } x = 0, \ \theta(0) = \theta_a \qquad \frac{\theta_b}{\cosh mb}$$

$$h = \frac{k_s d}{4} \left[\frac{\operatorname{arccosh}\left(\theta_b / \theta_a \right)}{b} \right]^2$$

Figure 1.1 A Relation for the Heat Transfer Coefficient

$$Nu_d = \frac{hd}{k_f} \tag{1.1}$$

where Nu_d is the Nusselt number based on the spine diameter.

The Rayleigh number was determined from the product of the Grashof number and the Prandtl number which were calculated from the following:

$$Gr \equiv \frac{g \ \beta (T_s - T) \ d^3}{\nu^2}$$

$$Pr \equiv \frac{c_p \mu}{k_f}$$

$$Ra = GrPr \tag{1.2}$$

where again the subscript "d" infers a dependence on the spine diameter and where the symbols are as defined in the nomenclature list.

II. EXPERIMENTAL APPARATUS

Figure 2.1 displays the configuration of the experimental apparatus and the techniques used in data acquisition. The following paragraphs provide a more indepth description of the component parts.

A. TEST DISK

The disk, shown in Figures 2.2 and 2.3, was fabricated entirely of 2024-T4 aluminum plate, 6.35 cm in thickness. A circular disk of diameter equal to 22.86 cm was cut from this plate and fitted with an array of 16 cylindrical spines as the heat transfer surfaces. Six equally spaced radial holes were drilled around the perimeter of the disk for insertion of six 200 watt cartridge heaters which provided the heat input necessary to elevate the temperature of the spines. The heaters were powered by a controllable power source in order to attain a steady-state condition for data measurements. The disk was fully insulated with soft rubber refrigeration insulation (k_I = 0.15 w/m-K) to minimize heat losses from the disk surface due to both radiation and convection.

B. CYLINDRICAL SPINES

The spines, shown in Figures 2.3 and 2.4 were similarly constructed of the 2024-T4 aluminum. Two sizes of spines were used in data gathering (0.9525 cm diameter x 3.75 cm length and 1.27 cm diameter x 5.08 cm length) to provide a wider range of Rayleigh Numbers. Extension pieces, also made of the same aluminum, were necessary to extend the actual base of the heat transfer surface (the cylindrical

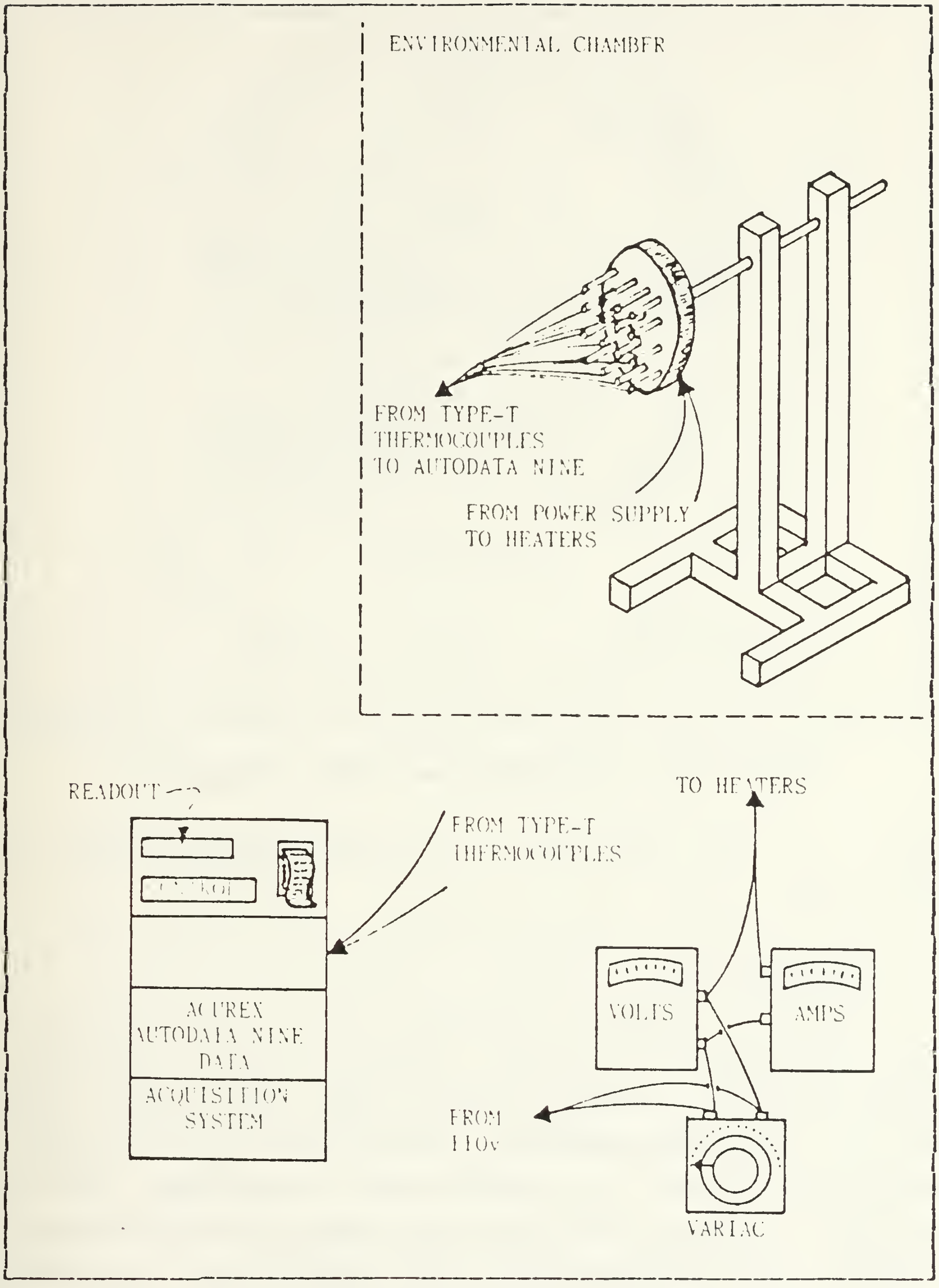


Figure 2.1 Sketch of the Test Apparatus and Configuration

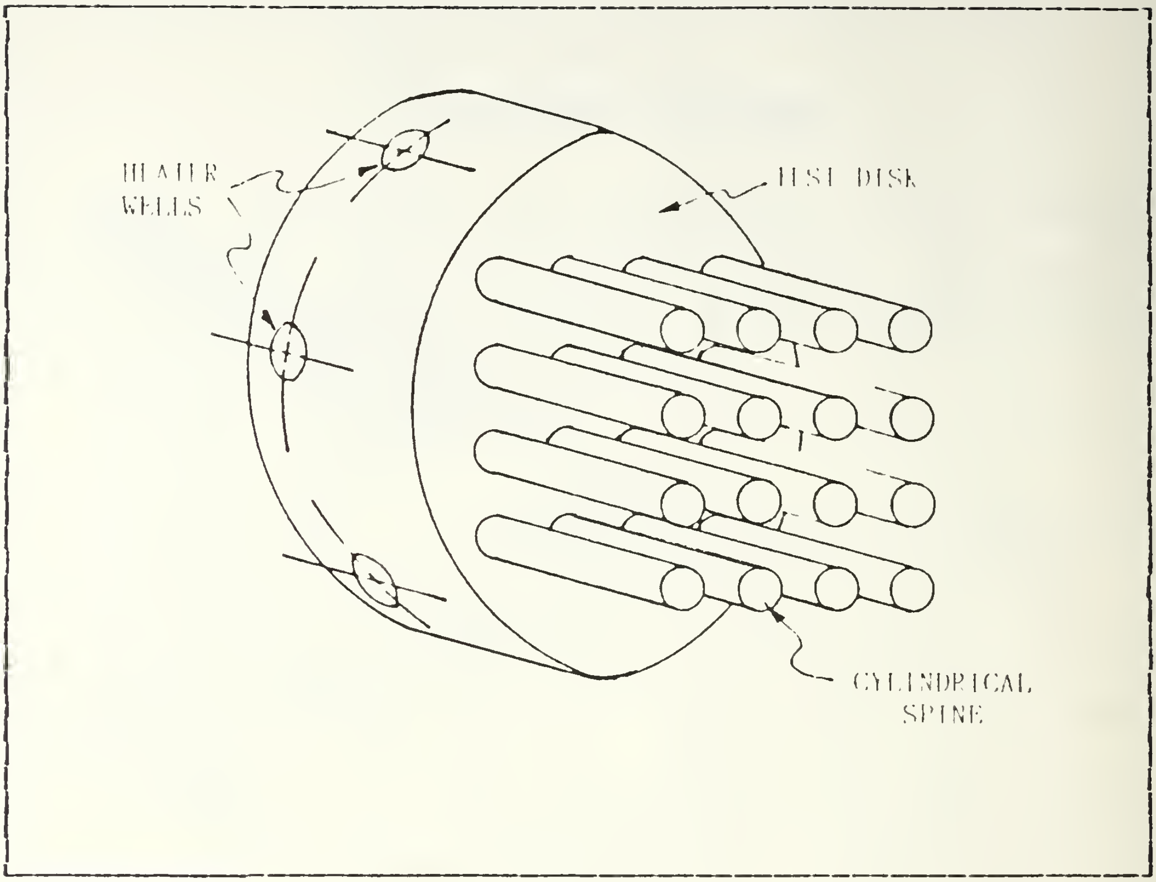


Figure 2.2 Test Disk and Spine Assembly

spine) to the outer level (exterior surface) of the insulation.

C. THERMOCOUPLES

All thermocouples were fabricated from copper-constantin (Type-T) of AWG No. 30 wire. They were butt welded with a spherical bead of approximately 0.05 cm diameter and smaller. All interconnecting cables were also of copper-constantin in order to minimize the uncertainty and to preclude, as much as possible, spurious thermoelectric effects. All thermocouples were connected to the ACUREX Autodata Nine data acquisition system for temperature recording.

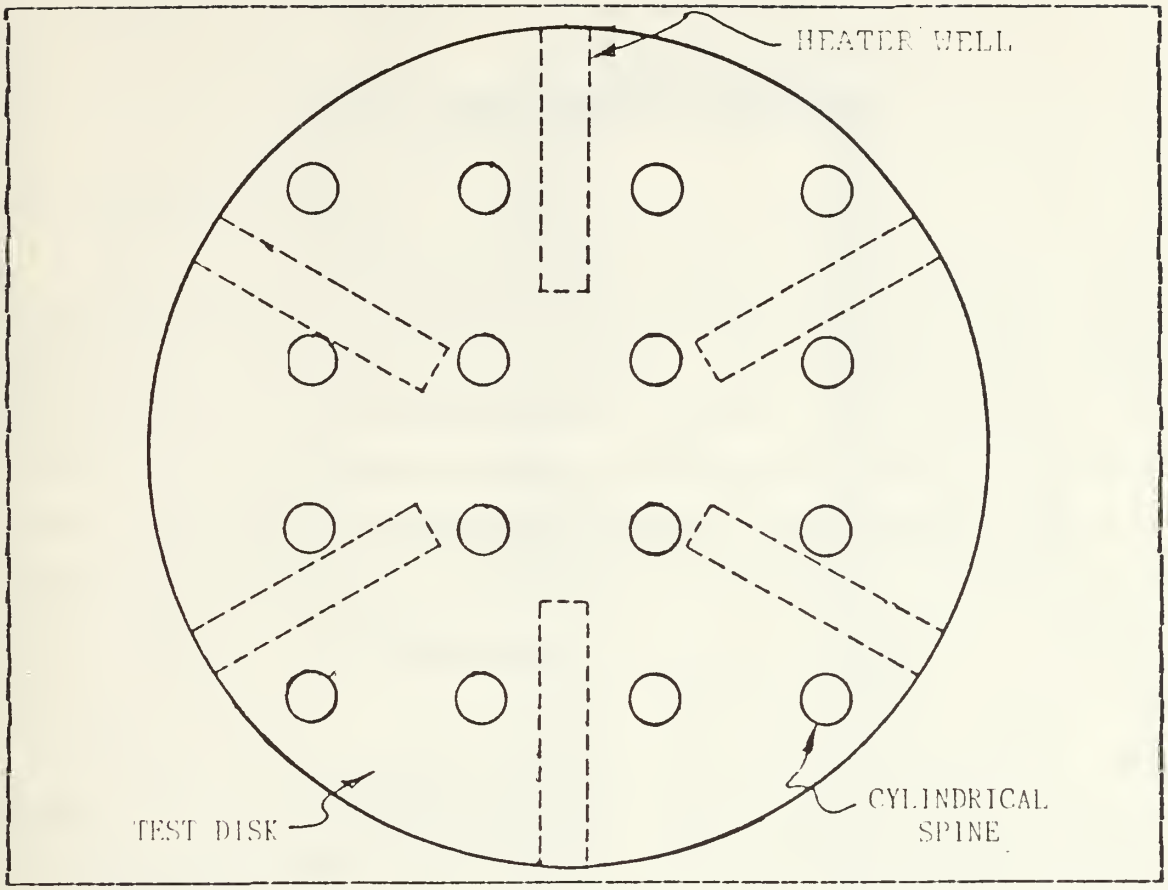


Figure 2.3 Sketch of Front View of Test Disk

D. ACUREX AUTODATA NINE

The thermocouple modules in the data acquisition system's thermocouple modules include an ice-point compensation network and provide automatic data scanning and printout for up to 100 channels of input. Due to the internal ice-point compensation networks, no external ice baths or reference junctions were required. In-place calibration checks were made at various temperatures over the expected temperature range in order to develop temperature correction factors for each thermocouple, its connecting cabling and the Autodata Nine circuitry as a unit. in addition, the complete disk and spine array was immersed in an ice-bath in order to establish one of the check points.

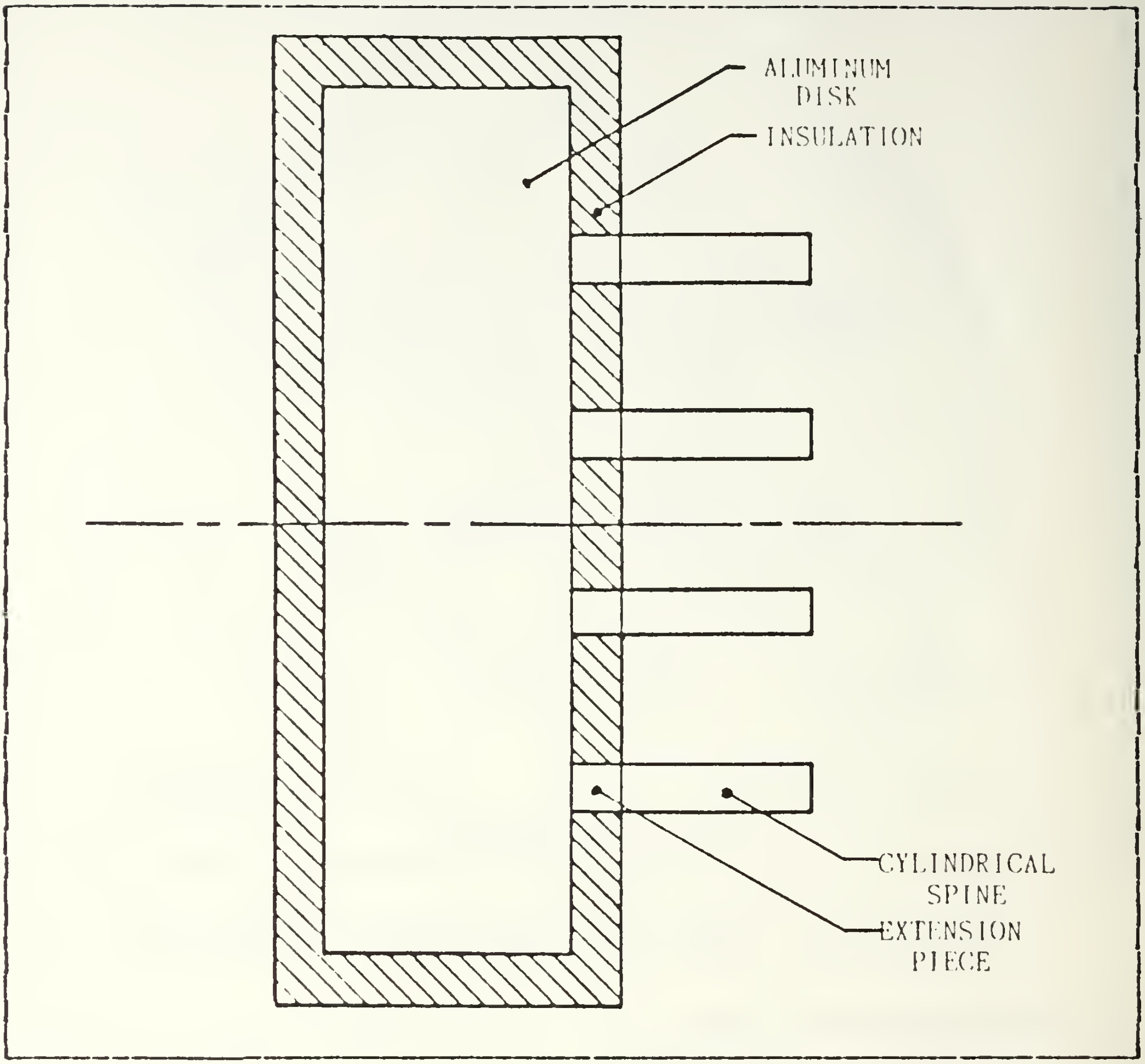


Figure 2.4 Test Disk and Spine Arrangement

E. ENVIRONMENTAL CHAMBER

The test system was housed in an environmental chamber in which the ambient temperatures could be maintained over a range between 35 and 135 degrees Fahrenheit.

III. EXPERIMENTAL PROCEDURE

A. DATA GATHERING

The experimental procedure was to set the disk at the desired roll angle using a liquid bubble level indicator and protractor to measure the angle. The temperature control thermostat of the environmental chamber was then set to the desired temperature. Steady state environmental conditions were attained in approximately six to eight hours. Next the heaters imbedded in the disk were energized to elevate the temperature of the disk and base of the spine to a temperature chosen for a predetermined Rayleigh number. The input voltage for the heaters was varied and the temperatures monitored until a steady state condition was reached. This was indicated by constant input power and constant temperatures. The time to attain a total steady-state condition was approximately three to five hours in addition to the time required for the chamber to stabilize.

A steady state condition was assumed to exist when no change in temperature occurred for four consecutive temperature readings taken at fifteen minute intervals. These temperatures were then used for the input data for the calculations. Typical dataruns are presented in Appendix B.

After completion of data taking at a set condition, changes were made in the roll angle, the temperature of the system or temperature of the environment, and the process of attaining and maintaining a new condition of steady state was again started.

B. DATA REDUCTION

Temperature data printed by the Autodata Nine during the individual runs was used as input parameters (corrected from the calibration curve for each thermocouple) for calculation of the desired parameters for plotting and evaluation.

All desired output parameters were calculated by the computer using appropriate relationships for the thermophysical properties [3] as functions of temperature and measured temperatures as input parameters. The computer program used for the calculations is provided in Appendix E.

Graphical display of the data was also utilized to aid in data analyses. These curves are shown as Figures B.1 through B.7.

IV. RESULTS AND CORRELATION

A. RESULTS

During each data collection run the temperatures at the base and tip of each spine, as well as four environmental temperatures were recorded. These temperatures were used as inputs for calculation of the various output parameters for each spine. These sixteen calculations were then averaged to provide the data point for that set of conditions. Forty seven data points were obtained. Typical data runs are included as Appendix C.

At a specified angle, the natural logarithm of the Nusselt number was plotted as a function of the natural logarithm of the Rayleigh number for each of the data points at a specified angle. These plots are included as Figures B.1 through B.5 in Appendix B. This graphical display was then used in order to facilitate correlation of the test data.

The plotted results indicated that a small dependence on roll angle did exist. Figures B.6 and B.7 were plotted to show these effects more specifically. Figure B.7 was plotted only for Rayleigh numbers between 579 and 586 because of the narrowness of the overall Rayleigh number range obtained.

B. CORRELATION

For each of the curves showing log (Nu) as a function of log (Ra), a least squares curve fit routine was used and a linear relation established to approximate these curves following a method presented by Kraus and Bar-Cohen[1]. A correlating equation was obtained for each of the roll angles ($\emptyset$), where $Z(\emptyset)$ defines a function of the roll angle.

The values of $Z(\phi)$ are tabulated in Table 1.

TABLE 1

Enhancement Factor

Roll Angle(ϕ)	Enhancement Factor ($Z(\phi)$)
0°	0.5916
10°	0.5986
20°	0.6075
30°	0.5990
45°	0.5928

The values of Nusselt numbers obtained from the experimental results are a function of a given Rayleigh number and roll angle and may be correlated via:

$$Nu_d = Z(\phi)Ra^{0.233} \tag{4.1}$$

However, observe that the dependence on the roll angle is weak (Table 1) and no mathematical relationship was determined for the values of $Z(\phi)$.

Results were also compared with several previously published correlations for the single horizontal cylinder. Churchill and Chu[6] suggested an equation of the form

$$Nu_d = \left\{0.60 + \frac{0.387\ Ra^{(1/6)}}{\left[1 + \left(\frac{0.559}{Pr}\right)^{(9/16)}\right]^{(8/27)}}\right\}^2$$

which correlated experimental results of several researchers very well.

The values obtained in this experimental program agreed favorably with these predictions with an average of 19% deviation throughtout the range investigated. Noting that the effect of heat transfer by radiation were also present, the deviation is attributed to radiation losses primarily. These losses would appear to not have a detrimental effect on the observed roll angle dependence.

V. CONCLUSION AND RECOMMENDATIONS

A. CONCLUSION

The results of the experimental program indicated that a dependence on roll angle did exist. This dependence is relatively minor (i.e., on the order of 3%), however for temperatures in the range of 200 degrees Fahrenheit this could mean a difference of 5 to 6 degrees in the surface temperature of an operating system. Therefore, further investigation at higher Rayleigh numbers may be deemed appropriate.

B. RECOMMENDATIONS

The following recommendations are provided for possible follow on projects of a similar nature.

1. Conduct the experimental program with the goal of determining and removing the radiation heat transfer.

2. Investigate other devices and methods of temperature measurement in order to remove more of the uncertainty associated with the use of thermocouples.

3. Conduct a similar experiment for the pitch angle.

4. Attempt a similar program with water as the convecting medium.

APPENDIX A

DEFINITION OF THE HEAT TRANSFER COEFFICIENT

Fins and spines (pin fins) of various geometries and thermal conductivities respond differently to identical and uniform heat sources and sinks. Similarly, there are numerous ways in which the temperatures and heat transfer coefficients of sources and sinks may vary. Important to the analysis of fin geometries are the constraints or assumptions which are employed to define and limit the problem and often to simplify its solution. The analysis of fins and spines employ the assumptions of Murray[7] and Gardner[8]. These limiting assumptions are:

1. The heat flow in the fin and its temperatures remain constant with time.
2. The fin material is homogeneous, its thermal conductivity is the same in all directions and it remains constant.
3. The heat transfer coefficient between the fin and the surrounding medium is constant and uniform over the entire surface of the fin (spine).
4. The temperature of the medium surrounding the fin is uniform.
5. The fin thickness is so small compared with its height that temperature gradients across the fin thickness may be neglected.
6. The temperature at the base of the fin is uniform.
7. There is no contact resistance where the base of the fin joins the prime surface.

8. There are no heat sources within the fin itself.

9. The heat transferred through the outermost edge of the fin is negligible compared with that leaving the fin through its lateral surface.

10. Heat transfer to or from the fin is proportional to the temperature excess between the fin and the surrounding medium.

In addition to proposing a profile function for longitudinal fins Gardner[8] proposed a profile function for spines:

$$f_2(x) = \frac{\delta_0}{2}\left(\frac{x}{b}\right)^{\frac{1-2n}{2-n}} x \qquad \text{(A.1)}$$

With the appropriate value of n, equation A.1 may be used for the development of a generalized differential equation for spines.

Figure A.1 shows a spine of arbitrary profile. It can be seen that the spine cross section normal to the flow of heat, the confining profile, and the perimeter of the spine are perfectly arbitrary functions of the distance x from the tip of the spine. A differential equation for the temperature excess, $\theta = t - t$, may be written by considering the heat conduction into and out of the element dx through the cross section f (x):

$$dq = k \frac{d}{dx}\left[f_1(x) \frac{d\theta}{dx}\right] dx$$

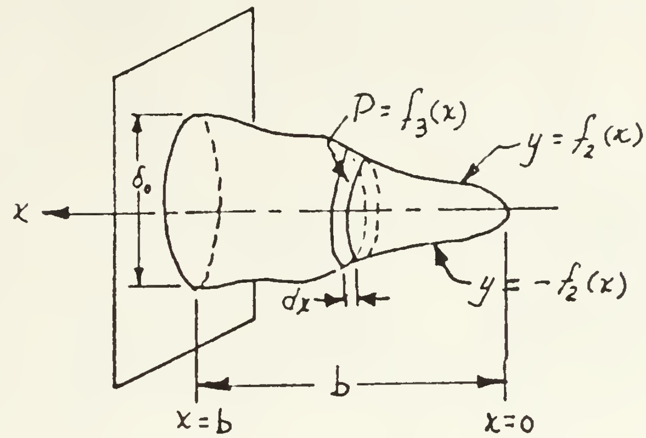


Figure A.1 Spine of Arbitrary Profile

The difference in heat flow into and out of the element must equal that dissipated by the surface of the spine. If dissipation occurs by convection and h is the convection coefficient

$$dq = h\, f_3(x)\, \theta\, dx$$

where $f_3(x)$ defines a perimeter function $P(x)$ which depends upon the distance x from the origin of the coordinate system. When convection and conduction are equated,

$$k\frac{d}{dx}\left[f_2(x)\frac{d\theta}{dx}\right] = hf_3(x)\theta$$

Upon rearrangement the generalized differential equation becomes

$$f_1(x)\,\frac{d^2\theta}{dx^2} + \frac{df_1(x)}{dx}\,\frac{d\theta}{dx} - \frac{h}{k}f_3(x) = 0 \qquad \text{(A.2)}$$

The relationship between $f_1(x)$ and $f_2(x)$ is

$$f_1(x) = \pi\left[f_2(x)\right]^2$$

where $f_2(x)$ was defined by equation A.1. Thus equation A.2 may be written

$$\left[f_2(x)\right]^2 \frac{d^2\theta}{dx^2} + \frac{d}{dx}\left[f_2(x)\right]^2 \frac{d\theta}{dx} - \frac{2h}{k} f_2(x)\theta = 0 \qquad \text{(A.3)}$$

where

$$f_3(x) = 2\pi f_2(x)$$

Equation A.3 is a second-order differential equation with variable coefficients except when the spine cross section normal to heat flow is constant. It may be solved by termwise comparison with the generalized Bessel equation.

For the cylindrical spine, shown in Figure A.2

$$f_2(x) = \frac{d}{2}$$

and

$$\frac{d}{dx}\left[f_2(x)\right]^2 = 0$$

For this case the exponent in equation A.1 has a value of zero corresponding to $n = 1/2$. Actually δ in equation A.1 is replaced by d, the spine diameter. Using this value of f_2

the general differential equation for the temperature excess becomes

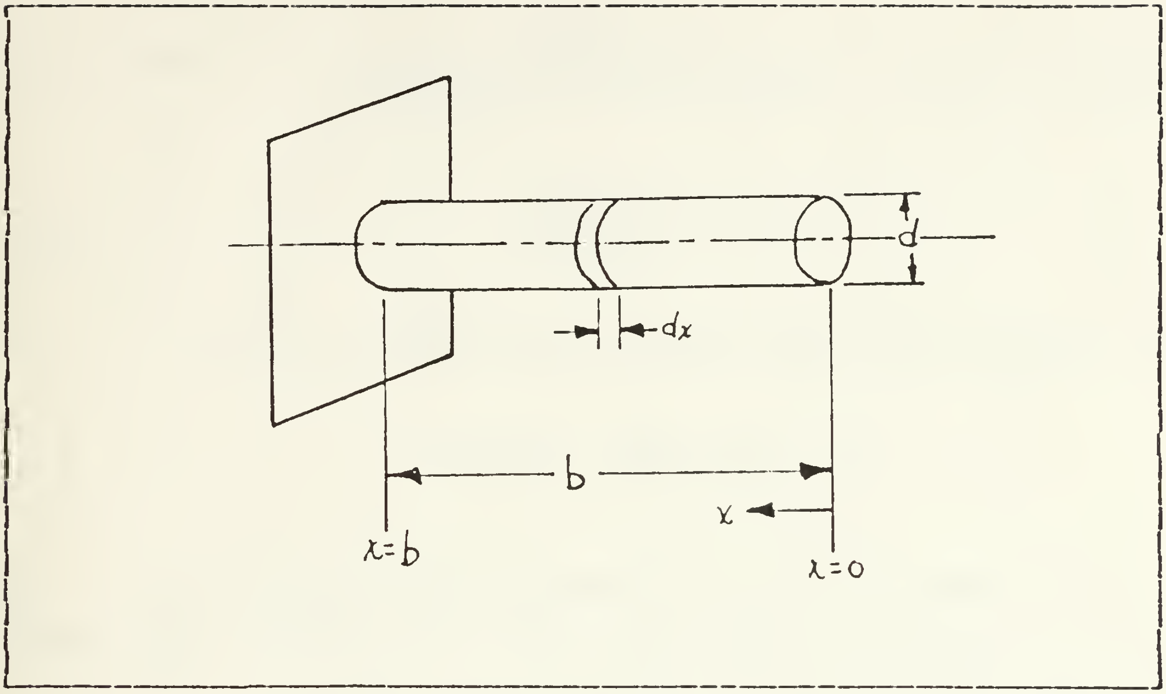


Figure A.2 Cylindrical Spine

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \tag{A.4}$$

where $m = (4h/kd)^{\frac{1}{2}}$.

The general solution to equation A.4 is

$$\theta(x) = C_1e^{mx} + C_2e^{-mx} \tag{A.5}$$

where the arbitrary constants are determined from the boundary conditions

$$\theta(x=b) = \theta_b \tag{A.6a}$$
$$\theta(x=0) = \theta_a \tag{A.6b}$$

This then leads directly to the equation for the temperature excess profile

$$\theta(x) = \theta_b \frac{\cosh mx}{\cosh mb} \tag{A.7}$$

and the heat flow entering the fin (spine) base is

$$q_b = \frac{\pi}{4} k d^2 m \theta_b \tanh(mb) \tag{A.8}$$

The value of h may be determined from equation (A.7) by taking temperature readings at the tip and the base which are precisely "b" meters apart.

Thus with base and tip temperature excesses,

$$\theta(x=b) = \theta_b, \theta(x=a) = \theta_a$$

It is easy to see from equation (A.7) that

$$\theta_a = \frac{\theta_b}{\cosh mb}$$

or that

$$\cosh(mb) = \frac{\theta_b}{\theta_a}$$

from which

$$mb = \operatorname{arccosh}(\theta_b/\theta_a) = \alpha \tag{A.9}$$

Then with $m = (4h/kd)^{1/2}$ one may then solve for h.

$$h = \frac{kd}{4}\left(\frac{\alpha}{b}\right)^2$$

APPENDIX B

GRAPHICAL DISPLAY OF DATA

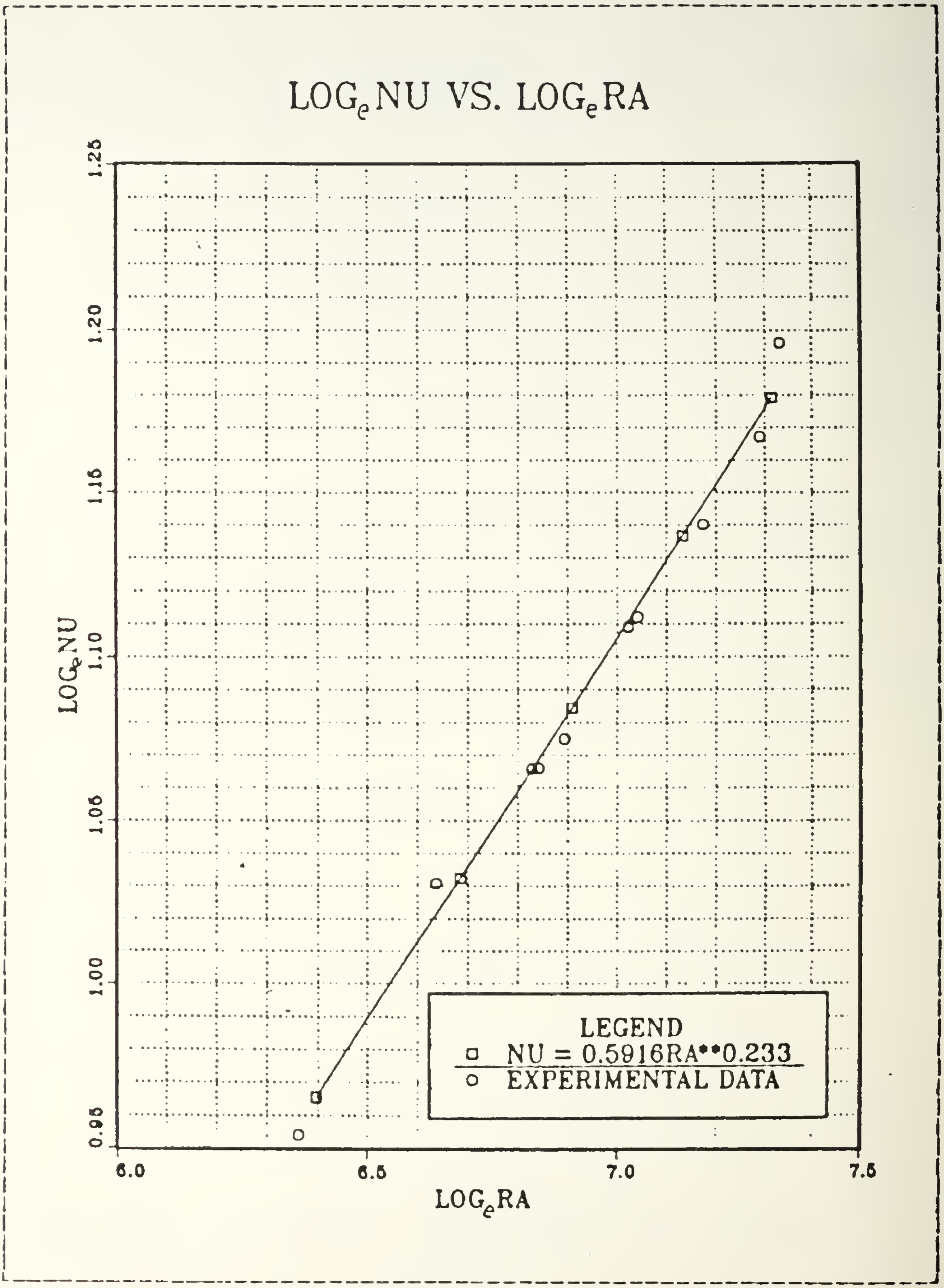


Figure B.1 $\log_e$ Nu vs. $\log_e$ Ra (0 degrees roll).

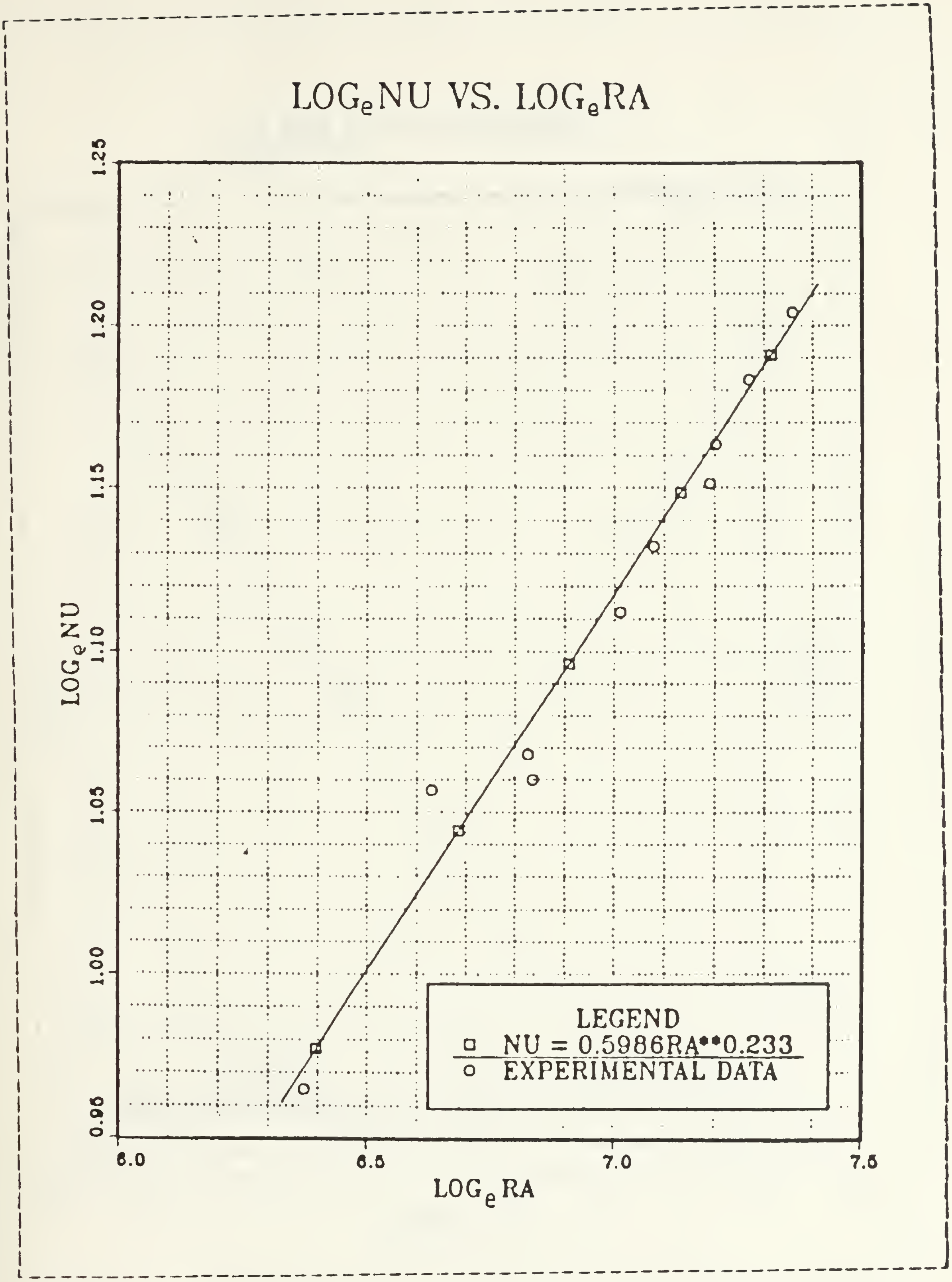


Figure B.2 log$_e$Nu vs. log$_e$Ra (10 degrees roll).

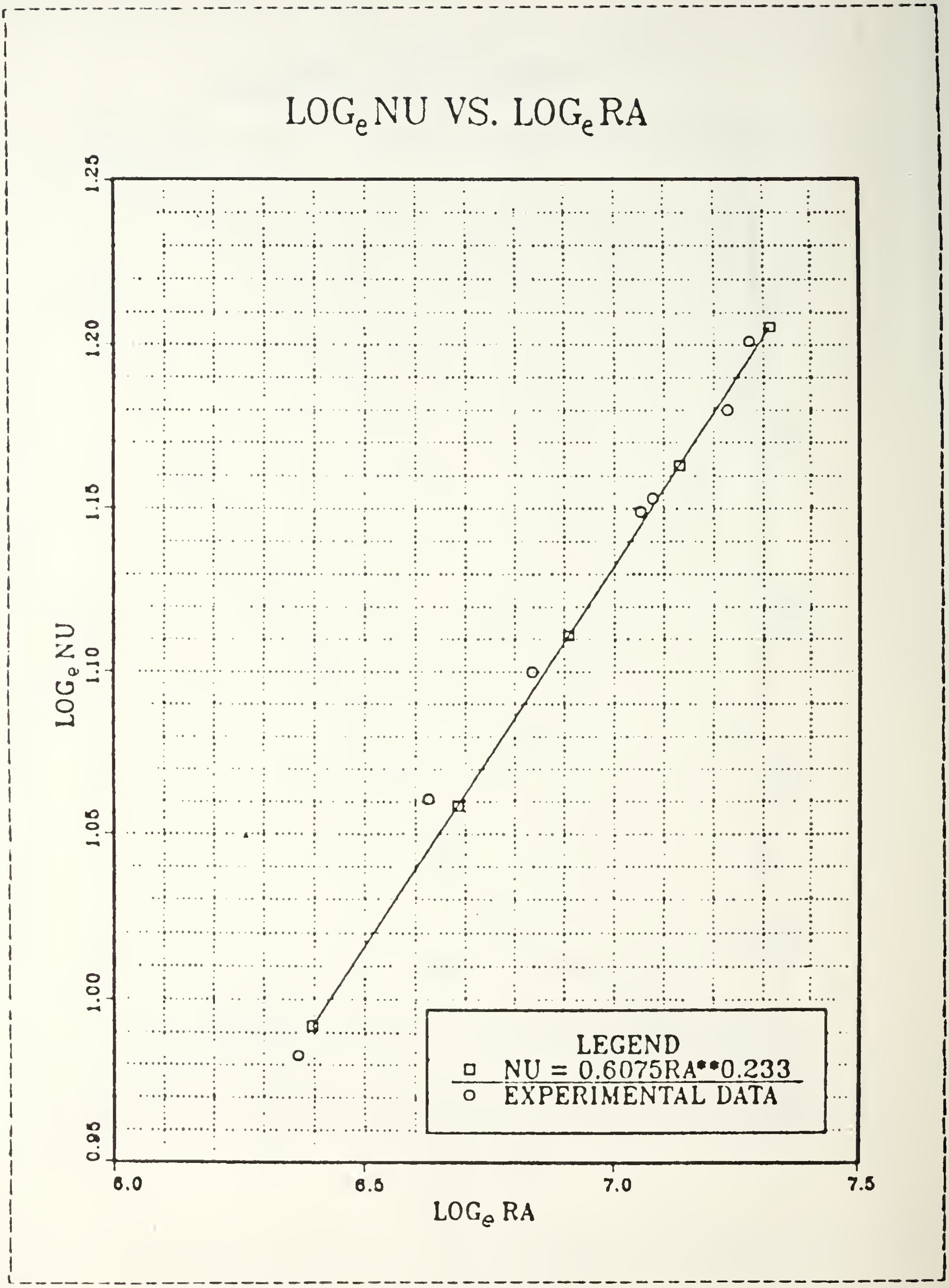


Figure B.3 $\log_e$Nu vs. $\log_e$Ra (20 degrees roll).

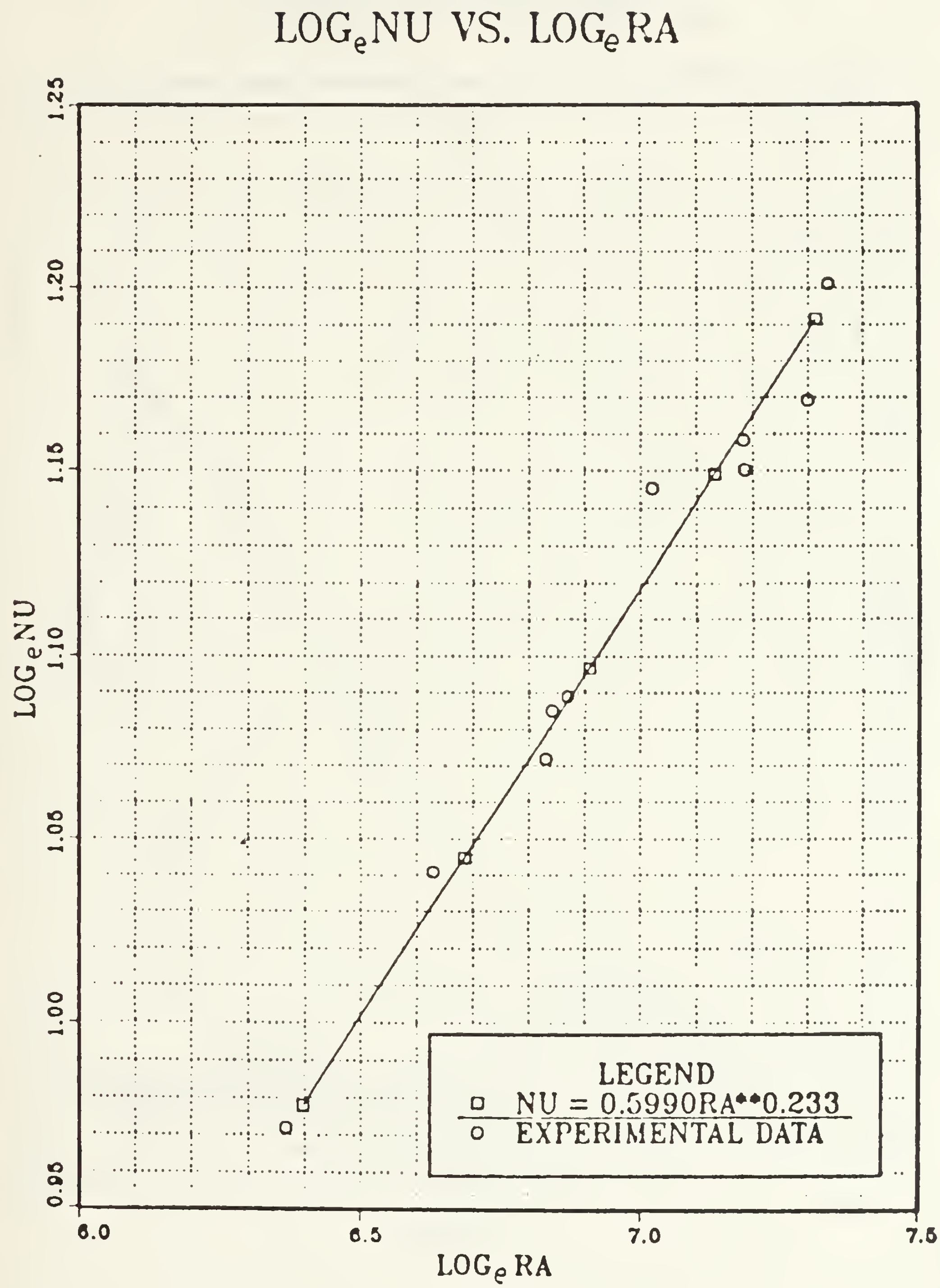


Figure B.4 log$_e$ Nu vs. log$_e$ Ra (30 degrees roll).

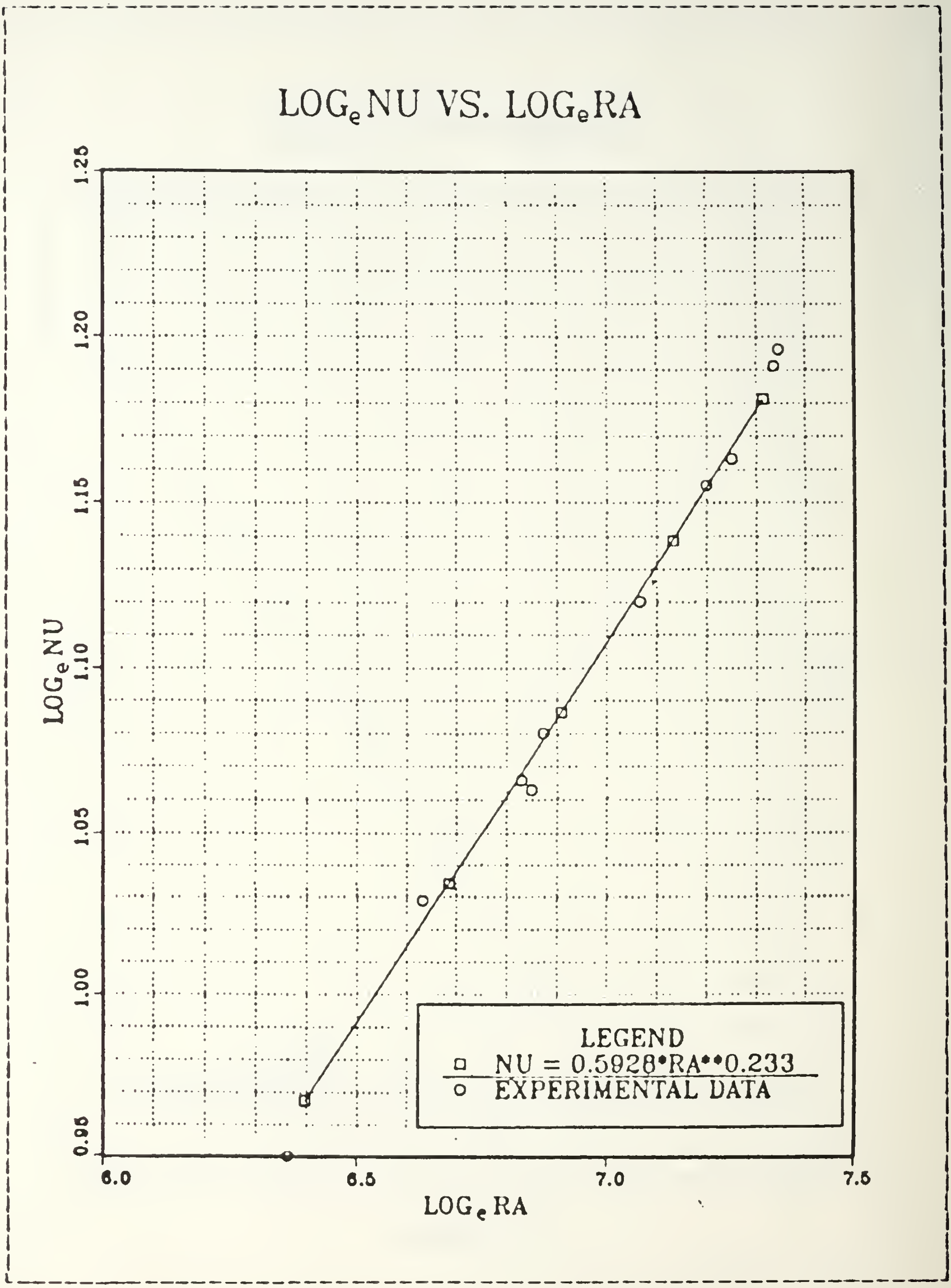


Figure B.5 $\log_e Nu$ vs. $\log_e Ra$ (45 degrees roll).

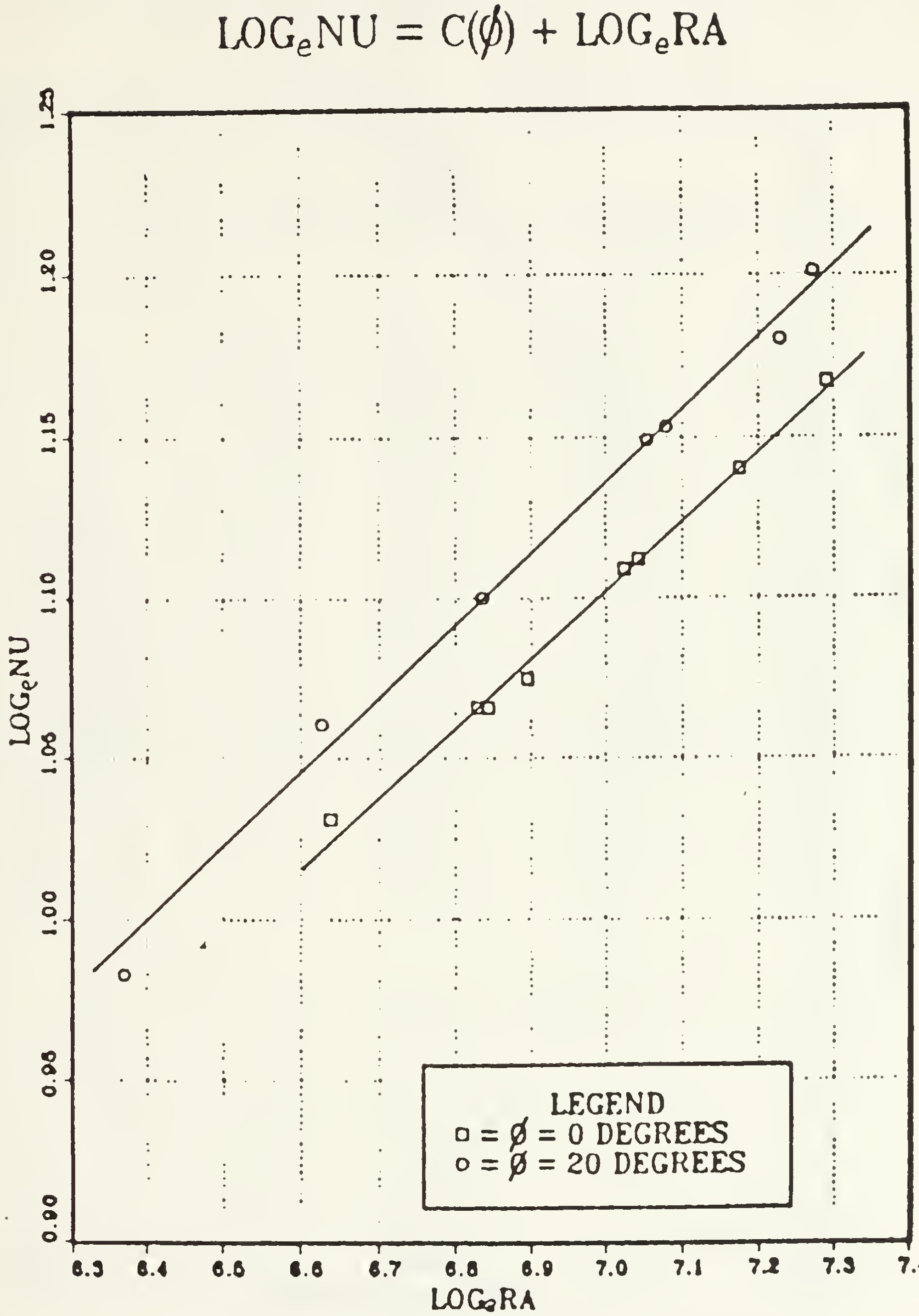


Figure B.6 Comparison of 0° and 20° Results.

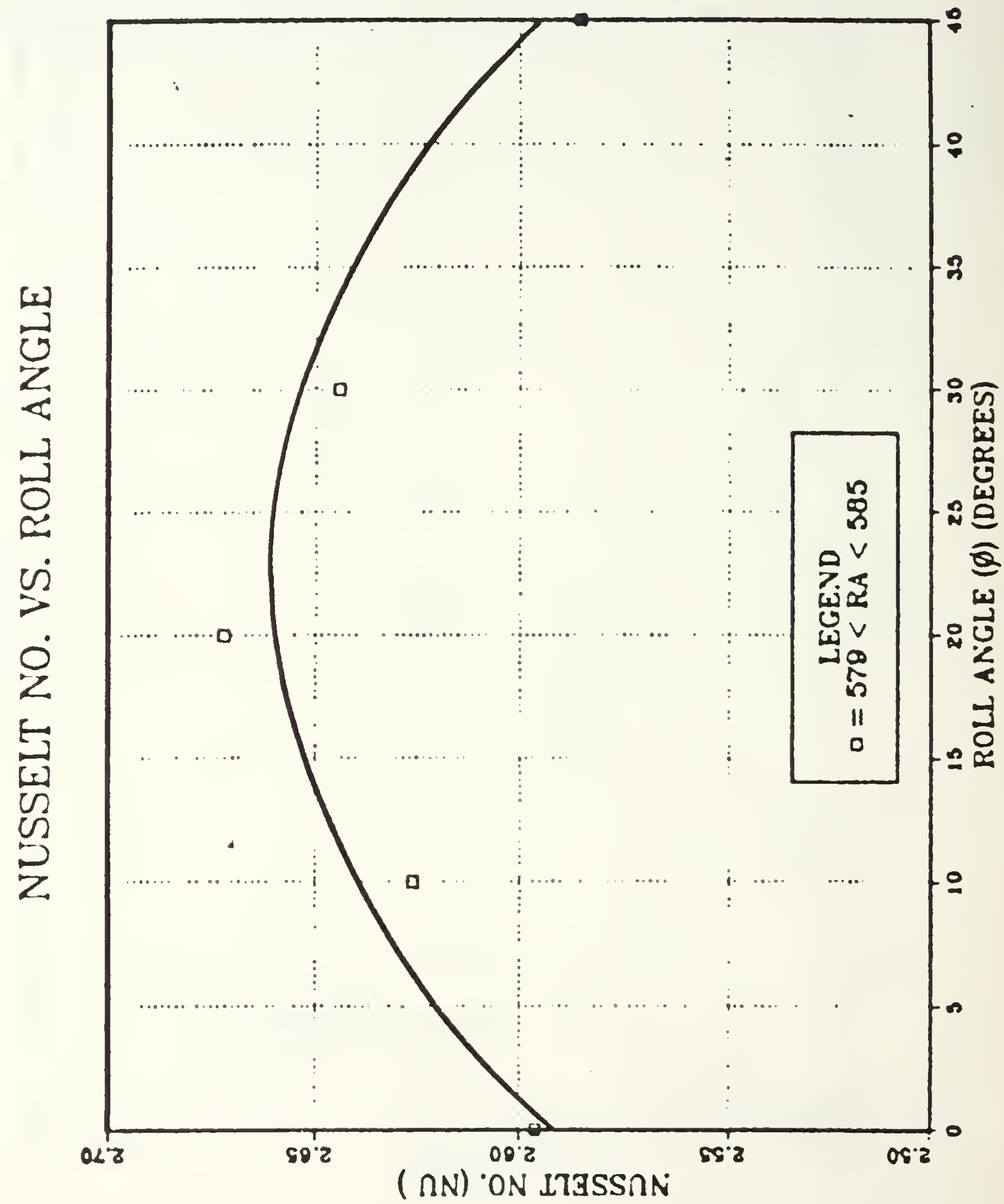


Figure B.7 Nu vs. Roll Angle (ϕ).

APPENDIX C

TYPICAL DATA RUNS

TABLE C.1

Typical Results (579 < Ra < 586)

T_s = 296.89   Ø = 0°

Pin	T_b(°K)	T_o(°K)	Ra	Nu	Nu_{ch}	h ($w/m^2 \cdot °K$)
1	312.48	312.25	579.40	2.2339	2.3333	6.162
2	312.32	312.05	579.31	2.5850	2.3283	7.128
3	312.50	312.23	579.40	2.9536	2.3333	7.043
4	312.32	312.03	572.94	2.8039	2.3280	7.731
5	312.37	312.08	574.69	2.7417	2.3295	7.560
6	312.59	312.33	582.43	2.4842	2.3358	6.853
7	312.61	312.35	583.17	2.4806	2.3364	6.843
8	312.64	312.34	583.84	2.7984	2.3367	7.720
9	312.41	312.16	576.73	2.4066	2.3311	6.637
10	312.44	312.16	577.28	2.7274	2.3316	7.522
11	312.58	312.31	581.97	2.6458	2.3354	7.298
12	312.66	312.38	584.27	2.5824	2.3373	7.124
13	312.60	312.32	582.43	2.6995	2.3358	7.446
14	312.37	312.14	575.71	2.2505	2.3303	6.206
15	312.54	312.25	580.31	2.7625	2.3340	7.620
16	312.48	312.19	578.48	2.7727	2.3326	7.647

T_s = 296.93   Ø = 10°

Pin	T_b(°K)	T_o(°K)	Ra	Nu	Nu_{ch}	h ($w/m^2 \cdot °K$)
1	312.76	312.47	586.28	2.7304	2.3389	7.534
2	312.59	312.33	581.14	2.5429	2.3347	7.015
3	312.78	312.51	587.20	2.5123	2.3396	6.932
4	312.59	312.31	580.77	2.7581	2.3344	7.609
5	312.64	312.36	582.51	2.6972	2.3358	7.441
6	312.81	312.55	588.37	2.4534	2.3406	6.770
7	312.89	312.63	590.95	2.4434	2.3426	6.743
8	312.86	312.57	589.49	2.7638	2.3415	7.627
9	312.63	312.38	582.70	2.3763	2.3360	6.556
10	312.67	312.38	583.25	2.6932	2.3364	7.430
11	312.86	312.58	589.76	2.6057	2.3417	7.191
12	312.87	312.60	588.37	2.5903	2.3420	7.038
13	312.82	312.54	590.21	2.6658	2.3406	7.356
14	312.59	312.31	580.77	2.7581	2.3344	7.609
15	312.76	312.47	580.25	2.7304	2.3389	7.534
16	312.71	312.42	584.44	2.7380	2.3374	7.554

TABLE C.1

Typical Results (579 $<$ Ra $<$ 586) (cont'd.)

T_s = 296.93 $\emptyset$ = 20°

1	312.71	312.42	584.44	2.7380	2.3374	7.554
2	312.54	312.27	579.29	2.5523	2.3332	7.040
3	312.72	312.46	585.36	2.5216	2.3381	6.958
4	312.54	312.25	578.92	2.7684	2.3329	7.636
5	312.59	312.31	580.68	2.7047	2.3343	7.461
6	312.76	312.44	585.84	2.9965	2.3384	8.268
7	312.78	312.52	587.28	2.4811	2.3397	6.791
8	312.81	312.51	587.65	2.7738	2.3400	7.654
9	312.58	312.33	580.86	2.3851	2.3345	6.579
10	312.56	312.27	579.57	2.7130	2.3343	7.484
11	312.75	312.47	586.09	2.6246	2.3387	7.242
12	312.82	312.54	588.37	2.5597	2.3406	7.063
13	312.77	312.48	586.56	2.6733	2.3391	7.376
14	312.54	312.25	578.92	2.7680	2.3329	7.636
15	312.59	312.31	580.77	2.7584	2.3344	7.609
16	312.59	312.31	580.77	2.7584	2.3344	7.609

T_s = 296.94 $\emptyset$ = 30°

1	312.65	312.36	582.81	2.7480	2.3359	7.581
2	312.54	312.27	579.29	2.5523	2.3332	7.040
3	312.67	312.40	583.52	2.5332	2.3366	6.989
4	312.54	312.25	578.92	2.7684	2.3329	7.636
5	312.53	312.31	579.76	2.1768	2.3336	6.005
6	312.76	312.44	585.64	2.9965	2.3384	8.268
7	312.78	312.52	587.28	2.4611	2.3397	6.791
8	312.58	312.27	579.94	2.9250	2.3337	8.068
9	312.56	312.27	579.97	2.7130	2.3334	7.484
10	312.70	312.51	586.73	2.2509	2.3392	6.211
11	312.69	312.42	584.27	2.6842	2.3373	7.268
12	312.52	312.24	588.37	2.6599	2.3406	7.063
13	312.77	312.48	586.56	2.6733	2.3391	7.376
14	312.48	312.14	575.25	2.7890	2.3299	7.692
15	312.59	312.31	580.77	2.7581	2.3344	7.609
16	312.59	312.31	580.77	2.7581	2.3344	7.609

TABLE C.1

Typical Results (579 < Ra < 586) (cont'd.)

T_s = 296.96   Ø = 45°

1	312.65	312.42	582.12	2.21640	2.33550	6.115
2	312.48	312.22	576.06	2.56722	2.33057	7.081
3	312.67	312.40	582.12	2.53847	2.33550	7.004
4	312.48	312.19	575.69	2.78452	2.33024	7.681
5	312.53	312.25	577.44	2.72293	2.33169	7.511
6	312.76	312.49	585.15	2.46985	2.33796	6.815
7	312.78	312.52	585.89	2.46626	2.33856	6.806
8	312.75	312.51	585.35	2.25553	2.33812	6.224
9	312.58	312.33	579.47	2.39009	2.33335	6.594
10	312.50	312.22	576.34	2.72653	2.33079	7.521
11	312.69	312.42	582.86	2.63975	2.33611	7.283
12	312.82	312.54	586.99	2.56502	2.33945	7.078
13	312.71	312.43	583.31	2.69100	2.33647	7.425
14	312.43	312.14	573.85	2.79496	2.32875	7.709
15	312.59	312.31	579.38	2.76402	2.33328	7.625
16	312.59	312.31	579.38	2.76402	2.33328	7.625

TABLE C.2

Typical Results (1305 < Ra < 1380)

T_s=317.83   Ø = 0°

Pin	T_b(°K)	T_o(°K)	Ra	Nu	Nu_{ch}	h (w/m^2·°K)
1	338.49	338.04	1304.18	2.97416	2.76085	6.553
2	338.49	337.99	1302.95	3.27379	2.76029	7.213
3	338.52	338.06	1305.56	3.07919	2.76146	6.784
4	338.44	337.04	1300.20	3.28048	2.75905	7.227
5	338.53	338.05	1305.56	3.15390	2.76146	6.949
6	338.49	338.02	1303.57	3.12388	2.76057	6.882
7	338.52	338.05	1305.40	3.11699	2.76139	6.868
8	338.50	338.01	1303.72	3.23238	2.76064	7.122
9	338.53	338.08	1306.47	3.00352	2.76188	6.618
10	338.53	338.08	1306.31	2.96674	2.76180	6.537
11	338.67	338.21	1313.81	2.98347	2.76517	6.575
12	338.63	338.17	1311.66	3.06200	2.76421	6.747
13	338.50	333.01	1303.73	3.23238	2.76064	7.122
14	338.51	338.04	1304.78	3.04564	2.76111	6.710
15	338.55	338.07	1306.63	3.18820	2.76195	7.025
16	338.51	338.01	1304.02	3.30642	2.76077	7.285

T_s=314.72   Ø = 10°

Pin	T_b(°K)	T_o(°K)	Ra	Nu	Nu_{ch}	h (w/m^2·°K)
1	334.82	334.35	1326.67	3.23972	2.77102	7.075
2	334.87	334.42	1330.20	3.07632	2.77259	6.718
3	334.81	334.33	1325.54	3.28189	2.77052	7.166
4	334.79	334.35	1325.88	3.04951	2.77067	6.659
5	334.76	334.32	1323.93	3.97872	2.76980	6.504
6	334.82	334.38	1327.63	3.00764	2.77145	6.568
7	334.75	334.28	1322.49	3.24195	2.76916	7.101
8	334.88	334.39	1329.72	3.34543	2.77238	7.306
9	334.87	334.42	1330.06	3.03809	2.77253	6.635
10	334.81	334.37	1326.84	3.05689	2.77110	6.654
11	334.88	334.41	1329.89	3.23042	2.77245	7.055
12	334.92	334.41	1331.16	3.45717	2.77302	7.550
13	334.95	334.49	1334.06	3.06572	2.77430	6.696
14	334.80	334.34	1325.88	3.12721	2.77067	6.829
15	334.93	334.47	1333.24	3.18207	2.77394	6.950
16	334.80	334.33	1325.54	3.20581	2.77052	7.000

TABLE C.2

Typical Results (1305 < Ra < 1380) (Cont.)

T_s=311.24   Ø = 10°

1	330.59	330.16	1347.11	3.11643	2.78019	6.736
2	330.48	330.07	1340.81	3.01289	2.77741	6.512
3	330.48	330.04	1339.97	3.21805	2.77704	6.955
4	330.43	329.98	1336.56	3.22605	2.77553	6.972
5	330.59	330.14	1346.60	3.23823	2.77997	7.000
6	330.54	330.07	1342.69	3.37037	2.77824	7.285
7	330.43	330.00	1337.07	3.10318	2.77576	6.706
8	330.59	330.16	1347.11	3.11653	2.78019	6.736
9	330.54	330.09	1343.38	3.20669	2.77855	6.931
10	330.43	329.98	1336.56	3.22605	2.77553	6.972
11	330.59	330.14	1346.60	3.23823	2.77997	7.000
12	330.60	330.14	1346.60	3.31969	2.77997	7.176
13	330.53	330.08	1342.87	3.25068	2.77832	7.026
14	330.48	330.05	1340.30	3.13528	2.77719	6.776
15	330.59	330.16	1346.93	3.15756	2.78012	6.825
16	330.37	329.92	1333.00	3.27927	2.77396	7.086

T_s=310.65   Ø = 20°

1	330.32	329.88	1377.21	3.06971	2.79335	6.628
2	330.32	329.89	1377.54	2.98876	2.79350	6.453
3	330.26	329.82	1373.47	3.16018	2.79173	6.823
4	330.26	329.82	1373.47	3.16018	2.79173	6.823
5	330.32	329.87	1376.70	3.18968	2.79313	6.887
6	330.26	329.80	1372.95	3.28058	2.79151	7.082
7	330.26	329.78	1372.26	3.44194	2.79121	7.431
8	330.37	329.88	1378.91	3.46225	2.79409	7.476
9	330.43	329.93	1381.96	3.53134	2.79541	7.625
10	330.32	329.87	1376.88	3.14908	2.79321	6.799
11	330.43	329.98	1383.67	3.13060	2.79614	6.760
12	330.43	329.98	1383.85	3.16992	2.69622	6.845
13	330.42	329.93	1381.82	3.49360	2.79535	7.544
14	330.32	329.82	1375.17	3.55229	2.79247	7.669
15	330.43	329.98	1383.67	3.13060	2.79614	6.760
16	330.26	329.82	1373.47	3.16018	2.79173	6.823

TABLE C.2

Typical Results (1305 < Ra < 1380) (Cont.)

T_s=303.00 Ø = 30°

1	319.53	319.14	1317.29	3.31536	2.76727	6.983
2	319.53	319.18	1318.68	3.07108	2.76789	6.468
3	319.56	319.18	1319.86	3.25894	2.76841	6.864
4	319.48	319.09	1313.92	3.37431	2.76576	7.106
5	319.55	319.18	1319.48	3.16447	2.76824	6.665
6	319.47	319.13	1314.72	2.98398	2.76611	6.285
7	319.51	319.12	1315.73	3.31993	2.76657	6.992
8	319.52	319.14	1316.70	3.26802	2.76700	6.883
9	319.55	319.17	1319.06	3.26120	2.76806	6.869
10	319.49	319.12	1315.10	3.27261	2.76629	6.892
11	319.52	319.17	1317.88	2.97346	2.76753	6.263
12	319.44	319.09	1312.56	3.03882	2.76515	6.400
13	319.56	319.22	1321.04	2.96517	2.76894	6.246
14	319.54	391.17	1318.47	3.21405	2.76779	6.770
15	319.47	319.08	1312.95	3.42424	2.76532	7.211
16	319.57	319.22	1321.46	3.06151	2.76913	6.449

T_s=299.02 Ø = 30°

1	314.53	314.19	1320.50	3.10591	2.76886	6.462
2	314.53	314.20	1320.91	3.10715	2.76904	6.464
3	314.57	314.23	1323.44	3.20306	2.77017	6.664
4	314.57	314.24	1323.66	3.04478	2.77027	6.335
5	314.56	314.23	1322.81	3.04922	2.76989	6.344
6	314.47	314.14	1316.47	3.06411	2.76706	6.374
7	314.52	314.18	1319.64	3.10825	2.76848	6.466
8	314.56	314.23	1323.03	3.09906	2.76999	6.448
9	314.61	314.29	1327.46	2.93205	2.77196	6.101
10	314.50	314.18	1319.00	2.9540s	2.76819	6.146
11	314.56	314.23	1323.25	3.04584	2.77009	6.337
12	314.51	314.14	1317.51	3.42918	2.76752	7.134
13	314.49	314.12	1316.02	3.48656	2.76686	7.253
14	314.41	314.15	1318.15	3.37434	2.76781	7.020
15	314.47	314.10	1314.57	3.43809	2.76621	7.152
16	314.53	314.20	1320.91	3.10715	2.76904	6.464

TABLE C.2

Typical Results (1305 < Ra < 1380) (Cont.)

T_s=301.86   Ø = 30.0°

1	318.91	318.58	1378.97	2.79324	2.79445	5.869
2	318.87	318.48	1373.41	3.27655	2.79204	6.884
3	318.96	318.56	1379.78	3.35353	2.79480	7.047
4	318.96	318.57	1379.99	3.21071	2.79489	6.747
5	319.00	318.62	1383.17	3.20213	2.79627	6.729
6	318.97	318.59	1381.18	3.20759	2.79540	6.740
7	319.02	318.63	1384.15	3.24665	2.79669	6.823
8	318.95	318.56	1379.36	3.25983	2.79462	6.850
9	318.89	318.52	1375.58	3.12968	2.79298	6.576
10	318.94	318.57	1379.57	3.11910	2.79471	6.554
11	318.95	318.57	1379.57	3.21197	2.79471	6.749
12	319.01	318.58	1382.16	3.53414	2.79583	7.426
13	318.99	318.56	1380.76	3.58582	2.79522	7.535
14	318.96	318.59	1380.76	3.02125	2.79522	6.349
15	318.91	318.54	1377.40	3.07746	2.79377	6.466
16	318.98	318.59	1381.35	3.25438	2.79548	6.838

T_s=309.50   Ø = 45°

1	328.21	327.79	1339.79	3.07511	2.77703	6.611
2	328.26	327.83	1342.74	3.19372	2.77833	6.866
3	328.27	327.87	1343.98	2.97942	2.77887	6.406
4	328.16	327.72	1335.93	3.33687	2.77532	7.173
5	328.15	327.75	1336.61	3.00068	2.77562	6.450
6	328.21	327.84	1341.36	2.69455	2.77772	5.793
7	328.31	327.84	1341.36	2.77899	2.77772	5.974
8	328.31	327.93	1347.48	2.80415	2.78042	6.029
9	328.21	327.71	1337.14	3.71214	2.77585	7.980
10	328.09	327.71	1333.65	2.88105	2.77431	6.193
11	328.26	327.79	1341.36	3.44784	2.77772	7.412
12	328.26	327.79	1341.54	3.48963	2.77780	7.502
13	328.19	327.69	1336.27	3.75752	2.77547	8.077
14	328.11	327.72	1334.52	2.92132	2.77469	6.279
15	328.26	327.83	1342.59	3.23456	2.77826	6.954
16	328.21	327.75	1338.55	3.45635	2.77648	7.430

APPENDIX D

DATA REDUCTION PROGRAM

```
C
C   ***********************************************************
C   *  PROGRAM TO CALCULATE HEAT TRANSFER COEFFICIENT ON A    *
C   *                PINNED CIRCULAR DISK                      *
C   ***********************************************************
C
C   ****************DEFINITION OF VARIABLES*****************
C    A     = FACTOR IN CHURCHILL AND CHU CORRELATION USED HERE
C            TO SIMPLIFY AND SHORTEN THE EXPRESSION
C    B     = LENGTH OF THE SPINE
C    C     = SIMPLIFYING FACTOR FOR CHURCHILL AND CHU CORRELA-
C            TION
C    CB    = CORRECTION TO THE SPINE BASE TEMPERATURE
C    CO    = CORRECTION TO THE SPINE TIP TEMPERATURE
C    D     = DIAMETER OF SPINE
C    E     = SIMPLIFYING FACTOR FOR THE CHURCHILL AND CHU COR-
C            RELATION
C    G     = GRAVITATIONAL ACCELERATION
C    GR    = GRASHOF NUMBER
C    H     = HEAT TRANSFER COEFFICIENT (EXPERIMENTAL)
C    KA    = THERMAL CONDUCTIVITY OF THE FLUID (AIR)
C    KP    = THERMAL CONDUCTIVITY OF THE SPINE MATERIAL
C    M     = A HEAT TRANSFER PROPERTY
C    NU    = NUSSELT NUMBER (FROM EXPERIMENTAL DATA)
C    NUC   = NUSSELT NUMBER (FROM CHURCHILL AND CHU CORRELA-
C            TION)
C    NUAVG = AVERAGE NUSSELT NUMBER (FROM EXPERIMENTAL DATA)
C    NUT   = TOTAL OF NUSSELT NUMBERS FROM 16 SPINES (1RUN)
C    PHI   = ROLL ANGLE (DEGREES)
C    PR    = PRANDTL NUMBER (EXPERIMENTAL)
C    R     = RATIO OF TEMPERATURE EXCESSES ( = THB/THO)
C    RA    = RAYLEIGH NUMBER (EXPERIMENTAL)
C    RAT   = TOTAL RAYLEIGH NUMBER FOR 16 SPINES (1 RUN)
C    RAV   = AVERAGE RAYLEIGH NUMBER (EXPERIMENTAL)
C    TB    = BASE TEMPERATURE (DEGREES F) UNCORRECTED
C    TO    = TIP TEMPERATURE (DEGREES F) UNCORRECTED
C    TBF   = CORRECTED BASE TEMP. (DEGREES F)
C    TOF   = CORRECTED TIP TEMP. (DEGREES F)
C    TB(I) = CORRECTED BASE TEMPERATURE (KELVIN)
C    TO(I) = CORRECTED TIP TEMPERATURE (KELVIN)
C    THB   = TEMPERATURE EXCESS AT THE BASE OF SPINE
C    THO   = TEMPERATURE EXCESS AT THE TIP OF SPINE
```

```
C     TF   = TEMPERATURE OF FILM BETWEEN SPINE AND FLUID
C     TS   = TEMPERATURE OF SURROUNDING FLUID (AIR)
C
C    ****************DECLARATION OF VARIABLES****************
      REAL D,KP,B,TS,R,M,H,PHI,NU,THB,THO,RAT,RAV,NUT,NUAVG,
      REAL A,C,E,G,TF,VA,ALFA,KA,GR,PR,RA,NUC
      REAL TB(25),TO(25),CB(25),CO(25)
      INTEGER I
C
C    **************INITIALIZATION OF VARIABLES***************
      D = 0.0127
      B = 0.0508
      KP = 120
      G = 9.807
      TS = 87.53
      PHI = 45
      NUT = 0
      RAT = 0
C
C    CALCULATE TS IN KELVIN
C
      TS = (TS - 32)*5/9 + 273.15
C
C    PRINT HEADINGS
      WRITE(6,100) TS,PHI
      WRITE(6,150)
C
C    READ IN AND CALCULATE CORRECTED TEMPERATURES, DEGREES F
C
      DO 50 I = 1,16
         READ(5,200)TB(I),TO(I),CB(I),CO(I)
         TBF = TB(I) + CB(I)
         TOF = TO(I) + CO(I)
C
C    CONVERT CORRECTED BASE & TIP TEMP. TO KELVIN
C
         TB(I) = (TBF - 32)*5/9 +273.15
         TO(I) = (TOF - 32)*5/9 +273.15
C
C    CALCULATE THETA BASE AND THETA ZERO (TEMP. EXCESSES)
C
         THB = TB(I) - TS
         THO = TO(I) - TS
C
C    CALCULATE RATIO OF THETA B TO THETA O (=cosh(MB))
C
         R = THB/THO
C
C    CALCULATE M
C
         M = (ALOG(R + SQRT((R**2)-1)))/B
C
```

```
C    CALCULATE HEAT TRANSFER COEFFICIENT H
C
         H = (M**2)*KP*D/4
C
C    CALCULATE FILM TEMPERATURE
C
         TF = (1./4.)*(TB(I) + TO(I)) + (1./2.)*TS
C
C    CALCULATE PROPERTIES OF AIR AT TF
C
         KA = 0.004033 + (7.30E-5)*TF
         VA = (-1.22567 + (9.350E-3)*TF)*(1.0E-5)
         ALFA = (-1.82333 + (0.01350*TF))*(1.0E-5)
C
C    CALCULATE NUSSELT NUMBER BASED ON EXPERIMENTAL H
C
         NU = H*D/KA
         NUT = NUT + NU
C
C    CALCULATE GRASHOF, PRANDTL, RAYLEIGH, AND NUSSELT NUM-
C    BERS BASED ON CHURCHILL & CHU CORRELATION
C
         PR = VA/ALFA
         GR = (G*(1.0/TF)*(TF-TS)*D**3)/VA**2
         RA = GR * PR
         A = 0.387*RA**(1.0/6.0)
         C = (1.0 + (0.559/PR)**(9.0/16.0))**(8.0/27.0)
         E = A/C
         NUC = (0.60 + E)**2
         RAT = RAT + RA
C    PRINT OUTPUT
C
         WRITE(6,250)I,TB(I),TO(I),RA,NU,NUC,H
50    CONTINUE
C
      RAV = RAT/16.0
      NUAVG = NUT/16.0
      WRITE(6,300)RAV,NUAVG
C
      STOP
100   FORMAT('1',15X,'TS=',F6.2,3X,'PHI=',F4.1)
150   FORMAT(17X,'PIN',4X,'TB',7X,'TO',7X,'RA',8X,'NU',8X,
     #     'NUC',7X,'H')
200   FORMAT(4(F 7.3))
250   FORMAT(18X,I2,2X,F6.2,3X,F6.2,2X,F7.2,2X,F8.5,2X,F8.5,
     #     3X,F6.3)
300   FORMAT(16X,'RAV =',F7.2,2X,'NUAVG =',F6.3)
C
      END
```

LIST OF REFERENCES

1. Kraus, A. D. and Bar-Cohen, A., Thermal Analysis and Control of Electronic Equipment, Hemisphere Publishing Co., New York, 1983.

2. Morgan, V. T., "The Overall Convective Heat Transfer from Smooth Circular Cylinders", Advances in Heat Transfer, vol. 11, D. P. Irvine and J. P. Hartnett, eds., Academic Press, New York, 1974.

3. Incropera, F. P. and DeWitt, D. P., Fundamentals of Heat Transfer, John Wiley and Sons, New York, 1981.

4. Schlichting, H., Boundary Layer Theory, 7th. ed., McGraw-Hill Book Company, 1979.

5. Kreith, P., Principles of Heat Transfer, 2nd. ed., International Textbook Co., Scranton,Pa., 1968.

6. Churchill, S. W. and Chu, H. H. S., "Correlating Equations for Laminar and Turbulent Free Convection from a Horizontal Cylinder", International Journal of Heat and Mass Transfer, vol. 18, pp. 1049-1053, 1975.

7. Murray, W. M., "Heat Dissipation Through an Annular Disk or Fin of Uniform Thickness", Journal of Applied Mechanics, vol. 5, p. A78, 1938.

8. Gardner, K. A., "Efficiency of Extended Surfaces", Transactions ASME, vol. 67, p. 621, 1945.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Technical Information Center Cameron Station Alexandria, Virginia 22314	2
2.	Library, Code 0142 Naval Postgraduate School Monterey, California 93943	2
3.	Department Chairman, Code 69 Department of Mechanical Engineering Naval Postgraduate School Monterey, California 93943	1
4.	Professor A.D. Kraus, Code 069KS Department of Mechanical Engineering Naval Postgraduate School Monterey, California 93943	5
5.	LCDR J.E. McClanahan 16206 Dartmoor Dr. Dumfries, Virginia 22026	2
6.	LCDR M. O'Hare 117 Brownell Circle Monterey, California 93940	1

Thesis
M1677 McClanahan
c.1 Evaluation of the
effects of roll angle
on the natural con-
vection heat transfer
from an array of cy-
lindrical spines.